Approximate Closed-Form Solution of the Synovial Pressure Field in the Human Ankle Joint with non-Newtonian Lubricant and Deformable Cartilage Layer

Alessandro Ruggiero [1], **Roberto D'amato** [2], **Emilio Gómez** [3], **Sergej Hloch** [4]

[1]Department of Industrial Engineering, University of Salerno. Italy
[2–3]Departamento de Mecánica Industrial -Universidad Politécnica de Madrid, Madrid. Spain
[4]Faculty of Manufacturing Technologies TUKE with a seat in Prešov, Slovak Republic
E-mail: [1]*ruggiero@unisa.it,* [2]*r.damato@upm.es,* [3]*emilio.gomez@upm.es,* [4]*sergej.hloch@tuke.sk*

Keywords: Hydrodynamic Lubrication, Analytical Model, Biotribology, Ankle Joint.

Introduction

The study of the synovial joints mechanism has recently become an active area of scientific research. The human joint is a dynamically loaded bearing [1, 2, 3] which employs articular cartilage as the bearing and synovial fluid as the lubricant. Once a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. Such joints have a low friction coefficient and negligible wear. However, when the natural joints fail to function properly, a possible remedy is the replacement of these by artificial joints.

During joint motion or when the cartilage is compressed, fluid content of the cartilage flows through the outermost layer of cartilage. The fluid flows back the moment the motion or compression is ceased. The moving fluid reduces friction and nourishes the blood vessel and cartilage. The human ankle joint can be simply taken as cylindrical [4], enabling rotation in the sagittal plane only. The joint is represented by two rigid circular cylinders in the inner contact (a cylinder encased in a cylindrical cavity), coated with thin layers of cartilage. The coupling model is assumed by two infinite rigid circular cylinders (subchondral bone) in the internal contact (a cylinder encased in a cylindrical cavity), covered with thin layer (articular cartilage) of uniform thickness; the lower (talar) articular surface is supposed stationary while the upper (tibial) surface is assumed to have pure squeeze motion $\dot{\varepsilon}(t)$ (Fig. 1).

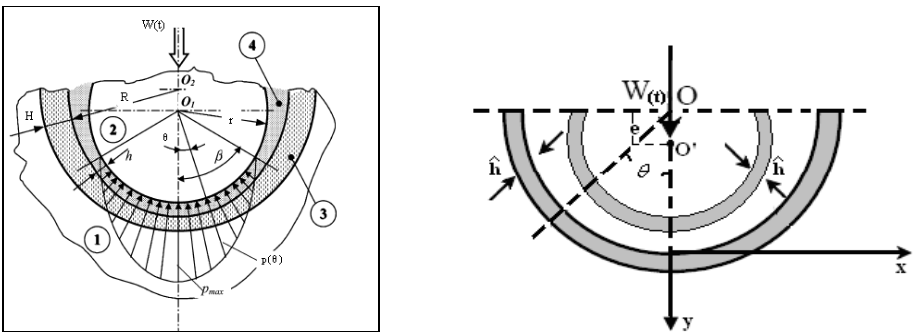

Fig. 1. a) Geometry of ankle joint. (1), (2)-Bone, (3)-cartilage layer, (4)-synovial fluid; b) Human ankle joint equivalent bearing

The long chain polysaccharide hyaluronic acid molecules present in the synovial fluid give us the motivation for modeling of the synovial fluid as a Stokes [3] couple-stress fluid.

Making the usual assumptions of hydrodynamic lubrication applicable to thin films [6], and considering the flow of a viscous fluid in a porous matrix governed by modified Darcy's law [3], the modified Reynolds equation, considering the nomenclature in Table 1, written in a non-dimensional form is:

$$\frac{\partial}{\partial\theta}\left\{\frac{\partial p^*}{\partial\theta}\left[f^*\left(h^*, l^*\right)+\frac{12\Phi^* H^*}{(1-\alpha)}\right]\right\}=12\cos(\theta). \quad (1)$$

As regards the determination of the thickness of the fluid film can be estimated that the same can be assessed as the sum of a component linked to the geometry of the system and another related to the elastic deformation of the thin coating cartilage, namely:

$$h = h_g + h_e. \quad (2)$$

Results and Conclusions

This approach gives the advantage to obtain an analytical expression of the synovial pressure field and of the non-stationary fluid film force acting in the synovial joint during the squeeze motion in terms of couple-stress parameter, film thickness, porosity and soft deformation parameters.

The calculated values for the pressure field and for the approximate fluid film force have been compared with those in [1, 2, 3] and they show a good agreement.

Table 1. Main nomenclature

e, ε	Eccentricity, Eccentricity ratio	*h*	Synovial film thickness	*p, p**	Pressure in the film region, Dimensionless pressure
R	Effective radius curvature of the contact	*H*	Porous layer thickness	*θ*	Circumferential coordinate
E	Young's modulus of the cartilage matrix	*l, l**	Couple stress parameter, dimensionless couple stress parameter	*ϕ*	Permeability of the cartilage matrix
*L, L**	Equivalent bearing length, Dimensionless bearing length	f_r	Synovial fluid film force	*μ*	Viscosity of synovial fluid

References

[1] **Walicki E., Walika A.** Mathematical modelling of some biological bearings. Smart Mater. Struct. 9 p. 280–283, (2000).

[2] **Ruggiero A., Gòmez E., D'Amato R.** Approximate Analytical Model for the Squeeze-Film Lubrication of the Human Ankle Joint with Synovial Fluid Filtrated by Articular Cartilage. TRIBOLOGY LETTERS, vol. Volume 41– N.2, p. 337–343, ISSN: 1023-8883, doi:10.1007/s11249-010-9710-5 (2010).

[3] **Ruggiero A., Gòmez E., D'Amato R.** Approximate closed-form solution of the synovial fluid film force in the human ankle joint with non-Newtonian lubricant. TRIBOLOGY INTERNATIONAL. Vol 57, p. 156–161, 2013, ISSN 0301-679X.

[4] **Walicki, E. and Walicka, A**. Inertia and couple-stress effects on squeeze-film characteristics with reference to biological bearings. Tribotest J., 2002, 8(3), 195–203.

[5] **Bujurke, N. M., Bhavi, S. G., and Naduvinamani, N. B.** The effect of couple stresses in squeeze film poroelastic bearings with special reference to synovial joints. IMA J. Math. Appl.Med. Biol., 1990, 7, 231–243.

[6] **Stokes,V. K.** Couple stresses in fluids. Phys. Fluids, 1966, 9, 1709.

[7] **Naduvinamani NB, Hiremath PS, Gurubasavaraj G.** Static and dynamic behaviour of squeeze film lubrication of narrow porous journal bearings with couple-stress fluid. Proc. Instn. Mech. Engrs, Part J 2001; 215: 45–62.

[8] **Lin J.-R. (1996),** Couple-stress effect on the squeeze film characteristics of hemispherical bearings with reference to synovial joints. – Appl. Mech. Eng., vol. 1, No. 2, p. 317–332.

[9] **Ruggiero A., Senatore A.** Approximate closed-form solution for the dynamical analysis of short bearings with couple stress fluid. Lubrication Science, vol. 19, p. 247–267, doi:10.1002/ls.47.