Research of stresses distribution in endoprosthesis contact considering the law of stochastic distribution of micro-roughness

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Introduction

Reliability and durability of endoprostheses work depend on the wear speed of its contact surfaces and on the material properties of the ball and the acetabular [1]. The speed of the wear mainly depends on pressure value in the contact surfaces and the value of the friction coefficient. The wear were analysed in some scientific papers [2–4]. However in those scientific papers pressure or stress distribution in the endoprostheses contact surfaces depending on the gap size between the acetabular and the ball is not clearly investigate. There is a lack of scientific works devoted for the development of stochastic mathematical models [2], and presented models are highly complex. While analyzing contact stiffness and stress magnitude in the contact, most of the authors are stating that contact surface is equal to surrounding surface. Latter assumption does not reflect the reality. In fact the surrounding surface is larger than surrounded. Therefore, actual contact stiffness and stress magnitude in contact can differ from the calculated considering that the contacting surface matches surrounding surface.

The main goal of this research is to develop stochastic mathematical model that would enable to simply analyze the estimation regularities of random values distribution density of two contacting surfaces and to determine distribution of stresses while considering the influence of surface micro- roughness and detrition degree.

Methods and mathematical modelling

In order to calculate of force F_i (1) is necessary to determine of summary distribution density of contacting surfaces micro-roughness random values (2).

$$F_i = c_0 Q_i \left[\delta_i \int_0^{\delta_i} f(\Delta_s)\, d\Delta_s - \int_0^{\delta_i} \Delta_s f(\Delta_s)\, d\Delta_s \right], \tag{1}$$

where $\Delta_S = \Delta_{R_{ri}} + \Delta_{R_{gi}}$; $f(\Delta_s)$ – summary distribution density of contacting surfaces micro-roughness random values; δ_i – magnitude of deformation; c_0 – coefficient of proportionality that evaluates strength properties of endoprosthesis material.

$$f\left(\Delta_{R_r}\right) = \frac{1}{\Delta_{R_{ri}}} \quad \text{and} \quad f\left(\Delta_{R_g}\right) = \frac{1}{\Delta_{R_{gi}}}, \tag{2}$$

where $\Delta_{R_{ri}}$ and $\Delta_{R_{gi}}$ – values of micro-roughness of surrounding and surrounded surfaces, when $0 \lhd \Delta_{R_{ri}} \lhd\lhd \infty$ and $0 \lhd \Delta_{R_{gi}} \lhd\lhd \infty$. Applying composition method of distributions of independent random values, i.e. $\Delta_S = \Delta_{R_{ri}} + \Delta_{R_{gi}}$, we can write:

$$f(\Delta_s) = \frac{\partial [F(\Delta_s)]}{\partial \Delta_s}, \quad f(\Delta_s) = \iint f(\Delta_{R_r}) f(\Delta_{R_g}) d\Delta_{R_r} d\Delta_{R_g}, \quad \Delta_{R_{ri}} + \Delta_{R_{gi}} \lhd \Delta_s. \quad (3)$$

Results

Knowing, that $\sigma_{ij} = \frac{F_i}{Q}$ from (1) we get the expression:

$$\sigma_{ij} = c_0 \left[\delta_{ij} \int_{\delta_{ij1}}^{\delta_{ijz}} \frac{\Delta_s}{\Delta_{R_r0} \Delta_{R_g0}} d\Delta_s - \int_{\delta_{ij1}}^{\delta_{ijz}} \frac{\Delta_s^2}{\Delta_{R_r0} \Delta_{R_g0}} d\Delta_s \right], \quad (4)$$

where i^{th} approach of contacting surfaces, j_k, j_{k+p} – separated interval of p points $0 \le \delta_{ij} \le \Delta_s$ in deformation interval. Integral (4) can be replaced approximately with finite sum.

The variation of calculated deformation values in contact is shown in Fig. 1. Regularity of distribution of stresses in different points of contacting surfaces is shown in Fig. 2. From the Fig. 2 it can be seen that during the wear of endoprosthesis, i.e. the gap between surrounding and surrounded surfaces is increasing, maximal values of stresses increases considerably.

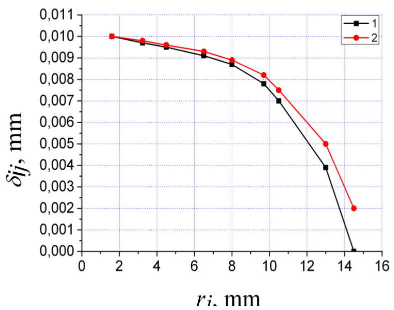

Fig. 1. Dependency of deformation values on the size of surrounding radius: 1 – the gap size between the acetabulum and the ball Δ = 0.01 mm; 2 – Δ = 0.005 mm

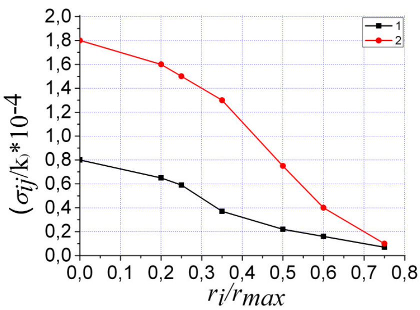

Fig. 2. Distribution of stress ratio in the contact surface of the endoprostheses when: 1 – the radius of the ball R_r =15 mm, Δ = 0,005 mm, δ_i = 0,001mm; 2 – R_r =15 mm, Δ =0,01 mm, δ_i = 0,001 mm

Conclusions

1. The worked out statistical mathematical model allows analyzing the regularities of estimation the law of density distribution of contacting surfaces random values. Evaluating summary micro- roughness distribution of two contacting surfaces estimated stress distribution depending on the wear, deformations and geometrical parameters.

2. The highest stress values are in the center of the contact surface of the endoprostheses. Their values are significantly higher than compared to those in the contact borders.
3. Stresses of the contacting surfaces increases with the increasing degree of surface wear and the maximal values can exceed several times the values for new endoprosthesis.

References

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