Pulse Wave Propagation in Arterial Beds of Upper Extremities: Comparison of 1d and 2d Models

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Abstract. Pulse wave propagation in the arterial systems of upper extremities is studied on nonlinear 1D and linearized 2D mathematical models. Three types of palm arches are considered. The pressure-flow curves in each artery are computed on 1D and 2D models and compared. It is shown the non-linear terms in 1D model lead to asymmetry of the pressure-flow curves, while the wave dispersion proper to the viscoelastic 2D model are negligible. Novel model-based approaches for the clinical pulse wave diagnostics are proposed.

Keywords: hemodynamics, arterial system, blood flow, pulse wave, medical diagnostics.

Introduction

Pulse wave analysis is a sensitive tool for early medical diagnostics [1, 2]. Anatomy of the arterial bed of upper extremity has some topological variations [3] that can influence pulse wave parameters [4]. The brachial artery (B) divides into radial (R) and ulnary (U) arteries that can be then connected by one A_1 or two $A_{1,2}$ arcs (Fig. 1a-c). Total impedances of the digital arteries are introduced as complex values Z_{1-4}. Since detailed geometry of the palmar arteries is individual and is used for personal identification, the studied problem is important. In the paper pulse wave propagation and reflection in the arterial beds without arcs (Fig. 1a), with one arc (Fig. 1b) and with two arcs (Fig. 1c) is studied on nonlinear 1D and linearized 2D models.

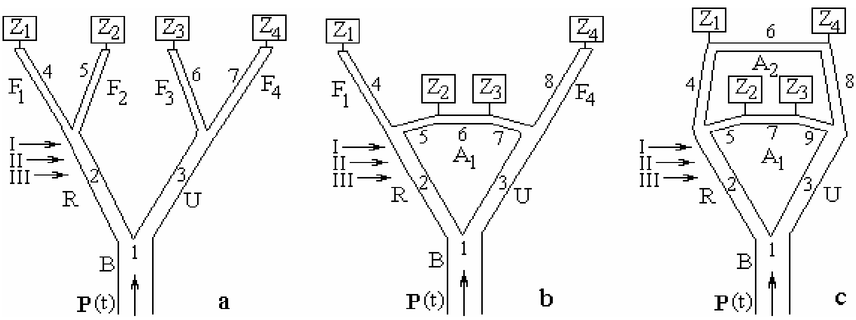

Fig. 1. Arterial systems of upper extremity without arcs (a), with one (b) and two (c) palmar arcs

Problem formulation and solution

Womersley model of 2D axisymmetric pulsatile flow is considered [1]. Each vessel is treated as a viscoelastic tube of length L_j, radius R_j and wall thickness h_j, where j is the serial number of the vessel in the complex branching system (Fig. 1a-c). Pressure $P_j(t,x_j)$ and volumetric rate $Q_j(t,x_j)$ in the j-th tube are introduced as [1]:

$$P_j(t,x_j) = P_j^0 e^{i\omega t}\left(e^{-i\omega x_j/c_j} + \Gamma_i e^{i\omega(x_j-2L_j)/c_j}\right), \tag{1}$$

where $Z_j^0 = \frac{\rho c_j}{\pi R_j^2}$, $c_j = \left(\frac{E_j h_j (1-F_j)}{2\rho R_j (1-\sigma_j^2)} e^{i\theta_j} \right)^{1/2}$, $F_j = \frac{2J_1(\beta_j)}{\beta_j J_0(\beta_j)}$, $P_j^0 = P_j \big|_{x_j=0}$, $\alpha_j = R_j \sqrt{\frac{\omega\rho}{\mu}}$, $\beta_j = \alpha_j i^{3/2}$, $i = \sqrt{-1}$, Γ_j are unknown reflection coefficients at the tubes' ends, $x_j \in [0, L_j]$ is the longitudinal coordinate along the tube number j, θ_j is dimensionless viscosity of the wall, ρ, μ are density and viscosity of the fluid, ω is angular frequency. Pressure and flow continuity conditions at each bifurcation are

$$Q_i(t, L_i) = Q_j(t,0) + Q_k(t,0), \qquad P_i(t, L_i) = P_j(t,0) = P_k(t,0) \tag{2}$$

where i, j, k are numbers of vessels in a bifurcation. At the inlet of the system $x_1 = 0$ the pressure wave $P_1(t,0) = P_1^0 e^{i\omega t}$ is given. From (1)–(2) one can obtain a nonlinear system of algebraic equations for Γ_j, P_j^0. For different geometrical models (Fig. 1a-c) the pressure distributions along $x_3 \in [0, L_3]$ have been calculated after numerical solution of the nonlinear system by modified Newton's method [4].

1D model is presented by Euler equations with additional drug term produced by viscosity and solved by the method of characteristics [5].

Conclusions

Calculations for (1)–(3) have been carries out at wide variations of Z_j, L_j, R_j and wall material properties E_j, θ_j in accordance with experimental data [1, 4, 5]. The position of the pulse wave diagnostic area at the patient's wrist has been chosen as $x_I = L_3 - l$, $x_{II} = L_3 - 2l$, $x_{III} = L_3 - 3l$, where $l = 1$ cm, the fluid and wall material properties as well as $\mathrm{Re}(Z_j)$, $\mathrm{Im}(Z_j)$ have been chosen within the physiological range [1].

For wide variations of P_1, ω no essential differences between the models a, b, c have been found, which is in agreement with [5]. Additional arches produces backward running wave and the superposition of the forward and backward waves increases the amplitudes of the pulse curves. The results substantiate the possibility of the pulse estimation without preliminary information on the individual structure of the arterial bed of upper extremity of a patient. Atherosclerosis, occlusions, blood insufficiency can be detected by comparative analysis of the pressure and flow waves measured on both arms.

References

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