11TH INTERNATIONAL CONFERENCE
BIOMDLORE 2016
OCTOBER 20–22, 2016, DRUSKININKAI, LITHUANIA
BIOMDLORE.VGTU.LT

eISSN 2345-0630
ISBN 978-609-457-959-2
eISBN 978-609-457-958-5
DOI: 10.3846/biomdlore.2016.05

Propagation of measurement uncertainty for balance platform model involving two output quantities

Adam Idźkowski[1], Wojciech Walendziuk[2], Aleksander Sawicki[3]
[1, 2, 3] Bialystok University of Technology, Poland
E-mails: [1] *a.idzkowski@pb.edu.pl (corresponding author)*, [2] *w.walendziuk@pb.edu.pl*,
[3] *aleksander.sawicki.91@gmail.com*

(Received 1 March 2016; accepted 22 June 2016)

Abstract. The paper topic is about the propagation of uncertainty in a multivariate measurement. The Law of Propagation of Uncertainty defined in the Guide to the Expression of Uncertainty in Measurement is used to calculate the standard uncertainties, covariance matrix and correlation coefficient. The procedure is presented for a set of samples, acquired from a balance platform, which are used to calculate the centre of pressure (*COP*) coordinates. The possibility of evaluating the uncertainty of *COP* results in different way is also discussed.

Keywords: balance platforms, centre of pressure, multivariate measurements, uncertainty.

Introduction

Multivariate measurements are present in many disciplines. Balance platforms are very popular devices and they are applied in rehabilitation or sport training. A training on a platform aims to stimulate parts of the human musculoskeletal system and the nervous system which are responsible i.a. for controlling the balance.

The ways of reliability and validity tests as well as accuracy calculations of the results obtained by using balance platforms are presented in many publications. The tests are performed in the following manner. The experimental method involves the use of additional testing devices intended to produce a concentrated force at a point of reference or force distributed over a specified area of the platform. In order to check the reliability, the arithmetic mean and standard deviation are calculated [1] as well as, among other parameters, the repeatability of series of measurements. A validation is performed by using another device with higher accuracy, which leads to determine the mean difference of results and the interclass correlation coefficient (*ICC*) [2], [3]. The authors of above mentioned publications did not assume that the coordinates of the *COP* were correlated. These parameters were determined for both coordinates COP_X, COP_Y separately. But they were, since they depended on the same input quantities. The aim of this paper is to present a bivariate measurement model and another way of uncertainty calculations basing on the data samples recorded by a balance platform.

Methods

It will be used a method for determining the uncertainty of measured values of *COP* components presented in the Guide ISO GUM [4]. This document assumes that the Law of Propagation of Uncertainty (LPU) should be used [5]. Output quantities *f* of a balance platform are two coordinates COP_X, COP_Y. Then the multivariate measurement model is

$$COP_X = \frac{L_X}{2}\frac{(TR+BR)-(TL+BL)}{TR+BR+TL+BL},\ COP_Y = \frac{L_Y}{2}\frac{(TR+TL)-(BR+BL)}{TR+BR+TL+BL}, \tag{1}$$

where: L_X, L_Y – dimensions of platform, weight values from four load sensors: *TL* – Top Right, *BL* – Bottom Right, *TR* – Top Right, *BR* – Bottom Right.

Input quantities *x* (*TR*, *BR*, *TL* and *BL*) and the equivalent sensors of a platform are independent what causes that covariance matrix *V* (*TR*, *BR*, *TL*, *BL*) is diagonal. A covariance matrix *V* (COP_X, COP_Y) is:

$$V(COP_X, COP_Y) \cong \left[\frac{\partial f}{\partial x}\right] \times V(TR, BR, TL, BL) \times \left[\frac{\partial f}{\partial x}\right]^T = \\ = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} = \begin{bmatrix} \sigma^2_{COP_X} & \mathrm{cov}(COP_X, COP_Y) \\ \mathrm{cov}(COP_Y, COP_X) & \sigma^2_{COP_Y} \end{bmatrix}. \qquad (2)$$

Jacobian matrix contains the partial derivatives of the scalar components of *f* with respect to the scalar elements of *x*. Then correlation coefficient is:

$$\rho_{xy} = \frac{\mathrm{cov}(COP_X, COP_Y)}{\sigma_{COP_X} \times \sigma_{COP_Y}} = \frac{V_{12}}{\sqrt{V_{11}} \times \sqrt{V_{22}}}. \qquad (3)$$

Results

The sample results in Tables 1 and 2 were acquired by putting a load on the Wii Balance Board (WBB). The WBB contains four transducers which are used to assess force distribution and the resultant movements in *COP* [3]. The platform was calibrated in the range 0–100 kg. The calculated parameters on the basis of covariance matrix are presented in Table 3.

Table 1. The experimental results of weight obtained for four sensors

Sample	*BL*[kg]	*BR* [kg]	*TL* [kg]	*TR* [kg]
1	3.3590	3.1680	4.0774	3.0436
….	…	…	…	…
300	3.3781	3.1302	4.0678	2.9465

Table 2. Standard deviations for 10, 100, 300 samples of the acquired *TR*, *BR*, *TL* and *BL*

Number of samples	σ_{BL} [kg]	σ_{BR} [kg]	σ_{TL} [kg]	σ_{TR} [kg]
10	0.0327	0.0205	0.0130	0.0344
100	0.0326	0.0294	0.0294	0.0281
300	0.0309	0.0299	0.0296	0.0305

Table 3. Combined uncertainties calculated for 10, 100, 300 samples (square roots from variances), their covariance coefficient (2) and correlation coefficient (3)

Number of samples	σ_{COPX} [cm]	σ_{COPY} [cm]	cov (COP_X, COP_Y)	Correlation coefficient
10	0.0061	0.0033	0.00001103	0.536
100	0.0068	0.0038	0.00000036	0.014
300	0.0069	0.0038	0.00000029	0.011

The result of measurement (i.e. 300 measured samples) can be presented as the mean and the surrounded area (ellipse) with 95% level of confidence (Fig. 1):

$$COP_X = (0.2979 \pm 0.0138)\ \text{cm}, \quad COP_Y = (0.1073 \pm 0.0076)\ \text{cm}. \qquad (4)$$

The results were verified by NIST Uncertainty Machine [6] which applies a univariate Monte Carlo Method.

After assuming Gaussian distributions of four input quantities it was created the summary statistics for 100000 realizations of the output quantity. The result was similar:

COP_X = (0.2952±0.0134) cm, COP_Y = (0.1100 ± 0.0070) cm for k = 2 and p = 0.95. (5)

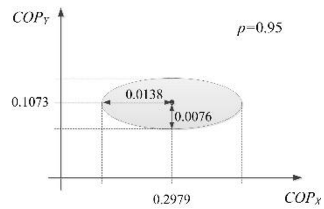

Fig. 1. Bivariate normal density contour (the result of COP with probability 95%)

The probability density functions (PDF) for both output quantities generated by NIST UM are presented in Fig. 2 and Fig. 3.

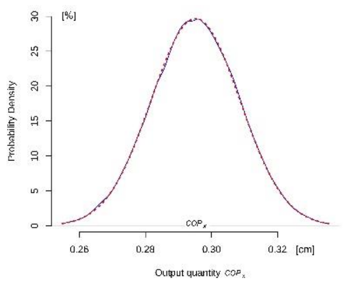

Fig. 2. Univariate PDF of COP_X

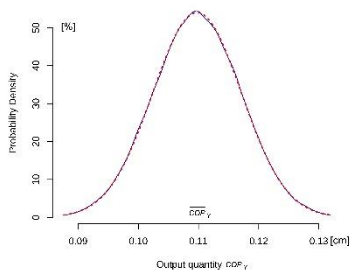

Fig. 3. Univariate PDF of COP_Y

Conclusions

The result (4) was obtained by using Law of Propagation of Uncertainty (LPU). The central region of a bivariate normal distribution is presented. For a condition $V_{11} > V_{22}$ and correlation coefficient near zero in (2) the axes of the ellipse are parallel to the coordinate axes, with the major axis parallel to the horizontal axis. For different conditions and correlation greater than 0 the ellipse can have a slope and can be elongated. The results (5) of simulation obtained by univariate Monte Carlo Method were convergent (for the same p = 0.95).

Acknowledgements

The paper was prepared at Bialystok University of Technology within a framework of the S/WE/1/2013 project sponsored by Ministry of Science and Higher Education.

References

[1] Dias, J. A., *et al.* 2011. Validity of a new stabilometric force platform for postural balance evaluation, *Brazilian Journal of Kinanthropometry and Human Performance* 13(5): 367–372. http://dx.doi.org/10.5007/1980-0037.2011v13n5p367

[2] Mauch, M.; Rist, H. J.; Kaelin, X. 2014. Reliability and validity of two measurement systems in the quantification of jump performance, *Schweizerische Zeitschrift für Sportmedizin und Sporttraumatologie* 62(1): 57–63.

[3] Clark, R. A., *et al.* 2010. Validity and reliability of the Nintendo Wii Balance Board for assessment of standing balance, *Gait & Posture* 31(3): 307–310. http://dx.doi.org/10.1016/j.gaitpost.2009.11.012

[4] *Evaluation of measurement data – supplement 2 to the "Guide to the expression of uncertainty in measurement" – Extension to any number of output quantities* [online]. 2011. Bureau International des Poids et Mesures [cited 1 March 2016]. Available from Internet: http://www.bipm.org/en/publications/guides/gum.html
[5] Hall, B. D. 2004. On the propagation of uncertainty in complex-valued quantities, *Metrologia* 41(3): 173–177. http://dx.doi.org/10.1088/0026-1394/41/3/010
[6] Lafarge, T.; Possolo, A. 2013. *NIST Uncertainty Machine – User's Manual* [online]. National Institute of Standards and Technology [cited 1 March 2016]. Available from Internet: http://uncertainty.nist.gov/NISTUncertaintyMachine-UserManual.pdf