Propagation of measurement uncertainty for balance platform model involving two output quantities

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Abstract. The paper topic is about the propagation of uncertainty in a multivariate measurement. The Law of Propagation of Uncertainty defined in the Guide to the Expression of Uncertainty in Measurement is used to calculate the standard uncertainties, covariance matrix and correlation coefficient. The procedure is presented for a set of samples, acquired from a balance platform, which are used to calculate the centre of pressure (COP) coordinates. The possibility of evaluating the uncertainty of COP results in different way is also discussed.

Keywords: balance platforms, centre of pressure, multivariate measurements, uncertainty.

Introduction

Multivariate measurements are present in many disciplines. Balance platforms are very popular devices and they are applied in rehabilitation or sport training. A training on a platform aims to stimulate parts of the human musculoskeletal system and the nervous system which are responsible i.a. for controlling the balance.

The ways of reliability and validity tests as well as accuracy calculations of the results obtained by using balance platforms are presented in many publications. The tests are performed in the following manner. The experimental method involves the use of additional testing devices intended to produce a concentrated force at a point of reference or force distributed over a specified area of the platform. In order to check the reliability, the arithmetic mean and standard deviation are calculated [1] as well as, among other parameters, the repeatability of series of measurements. A validation is performed by using another device with higher accuracy, which leads to determine the mean difference of results and the interclass correlation coefficient (ICC) [2], [3]. The authors of above mentioned publications did not assume that the coordinates of COP were correlated. These parameters were determined for both coordinates COPₓ, COPᵧ separately. But they were, since they depended on the same input quantities. The aim of this paper is to present a bivariate measurement model and another way of uncertainty calculations basing on the data samples recorded by a balance platform.

Methods

It will be used a method for determining the uncertainty of measured values of COP components presented in the Guide ISO GUM [4]. This document assumes that the Law of Propagation of Uncertainty (LPU) should be used [5]. Output quantities f of a balance platform are two coordinates COPₓ, COPᵧ. Then the multivariate measurement model is

\[
\begin{align*}
COP_x &= \frac{L_x}{2} \left( TL + BR - (TR + BL) \right), \\
COP_y &= \frac{L_y}{2} \left( TR + TL - (BR + BL) \right),
\end{align*}
\]

where: \(L_x, L_y\) – dimensions of platform, weight values from four load sensors: \(TL\) – Top Right, \(BL\) – Bottom Right, \(TR\) – Top Right, \(BR\) – Bottom Right.

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Input quantities $x$ ($TR$, $BR$, $TL$ and $BL$) and the equivalent sensors of a platform are independent what causes that covariance matrix $V$ ($TR$, $BR$, $TL$, $BL$) is diagonal. A covariance matrix $V(COP_x, COP_y)$ is:

$$V(COP_x, COP_y) \approx \left[ \frac{\partial f}{\partial x} \right] \times V(TR, BR, TL, BL) \times \left[ \frac{\partial f}{\partial x} \right]^T =$$

$$\begin{bmatrix}
V_{11} & V_{12} \\
V_{21} & V_{22}
\end{bmatrix}=\begin{bmatrix}
\sigma_{COP_x}^2 & \text{cov}(COP_x, COP_y) \\
\text{cov}(COP_y, COP_x) & \sigma_{COP_y}^2
\end{bmatrix} \tag{2}
$$

Jacobian matrix contains the partial derivatives of the scalar components of $f$ with respect to the scalar elements of $x$. Then correlation coefficient is:

$$\rho_{xy} = \frac{\text{cov}(COP_x, COP_y)}{\sigma_{COP_x} \times \sigma_{COP_y}} = \frac{V_{12}}{\sqrt{V_{11} \times V_{22}}} \tag{3}$$

Results

The sample results in Tables 1 and 2 were acquired by putting a load on the Wii Balance Board (WBB). The WBB contains four transducers which are used to assess force distribution and the resultant movements in $COP$ [3]. The platform was calibrated in the range 0–100 kg. The calculated parameters on the basis of covariance matrix are presented in Table 3.

Table 1. The experimental results of weight obtained for four sensors

<table>
<thead>
<tr>
<th>Sample</th>
<th>$BL$ [kg]</th>
<th>$BR$ [kg]</th>
<th>$TL$ [kg]</th>
<th>$TR$ [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.3590</td>
<td>3.1680</td>
<td>4.0774</td>
<td>3.0436</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>300</td>
<td>3.3781</td>
<td>3.1302</td>
<td>4.0678</td>
<td>2.9465</td>
</tr>
</tbody>
</table>

Table 2. Standard deviations for 10, 100, 300 samples of the acquired $TR$, $BR$, $TL$ and $BL$

<table>
<thead>
<tr>
<th>Number of samples</th>
<th>$\sigma_{BL}$ [kg]</th>
<th>$\sigma_{BR}$ [kg]</th>
<th>$\sigma_{TL}$ [kg]</th>
<th>$\sigma_{TR}$ [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0327</td>
<td>0.0205</td>
<td>0.0130</td>
<td>0.0344</td>
</tr>
<tr>
<td>100</td>
<td>0.0326</td>
<td>0.0294</td>
<td>0.0294</td>
<td>0.0281</td>
</tr>
<tr>
<td>300</td>
<td>0.0309</td>
<td>0.0299</td>
<td>0.0296</td>
<td>0.0305</td>
</tr>
</tbody>
</table>

Table 3. Combined uncertainties calculated for 10, 100, 300 samples (square roots from variances), their covariance coefficient (2) and correlation coefficient (3)

<table>
<thead>
<tr>
<th>Number of samples</th>
<th>$\sigma_{COP_x}$ [cm]</th>
<th>$\sigma_{COP_y}$ [cm]</th>
<th>$\text{cov} (COP_x, COP_y)$</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0061</td>
<td>0.0033</td>
<td>0.0000001103</td>
<td>0.536</td>
</tr>
<tr>
<td>100</td>
<td>0.0068</td>
<td>0.0038</td>
<td>0.00000036</td>
<td>0.014</td>
</tr>
<tr>
<td>300</td>
<td>0.0069</td>
<td>0.0038</td>
<td>0.00000029</td>
<td>0.011</td>
</tr>
</tbody>
</table>

The result of measurement (i.e. 300 measured samples) can be presented as the mean and the surrounded area (ellipse) with 95% level of confidence (Fig. 1):

$$COP_x = (0.2979 \pm 0.0138) \text{ cm}, \ COP_y = (0.1073 \pm 0.0076) \text{ cm}. \tag{4}$$

The results were verified by NIST Uncertainty Machine [6] which applies a univariate Monte Carlo Method.
After assuming Gaussian distributions of four input quantities it was created the summary
statistics for 100000 realizations of the output quantity. The result was similar:

\[
COP_X = (0.2952 \pm 0.0134) \text{ cm}, \quad COP_Y = (0.1100 \pm 0.0070) \text{ cm} \quad \text{for} \quad k = 2 \quad \text{and} \quad p = 0.95.
\] (5)

\[\begin{array}{c}
\text{Fig. 1. Bivariate normal density contour (the result of COP with probability 95%)}
\end{array}\]

The probability density functions (PDF) for both output quantities generated by NIST UM are
presented in Fig. 2 and Fig. 3.

\[\begin{array}{c}
\text{Fig. 2. Univariate PDF of } COP_X \\
\text{Fig. 3. Univariate PDF of } COP_Y
\end{array}\]

Conclusions
The result (4) was obtained by using Law of Propagation of Uncertainty (LPU). The central
region of a bivariate normal distribution is presented. For a condition \(V_{11} > V_{22}\) and correlation
coefficient near zero in (2) the axes of the ellipse are parallel to the coordinate axes, with the major
axis parallel to the horizontal axis. For different conditions and correlation greater than 0 the
ellipse can have a slope and can be elongated. The results (5) of simulation obtained by univariate
Monte Carlo Method were convergent (for the same \(p = 0.95\)).

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